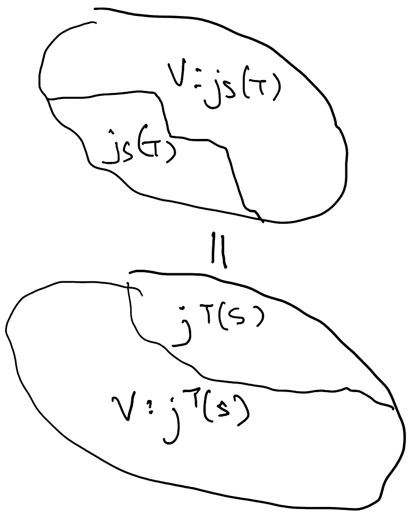
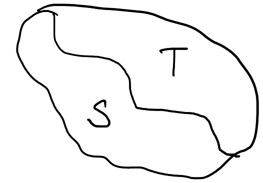
Math 249, Wednesday April 8

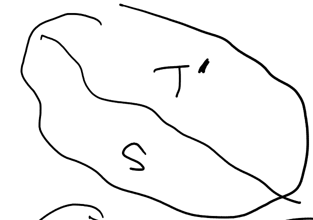
Switching lemma

$js(T)$, $V: js(T)$



$j^T(S)$, $V: j^T(S)$





$js(T) = V: j^T(S)$

$j^T(S) = V: js(T)$

$T \sim T'$

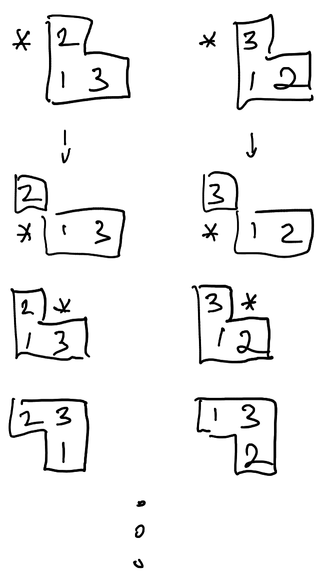
$\Downarrow$

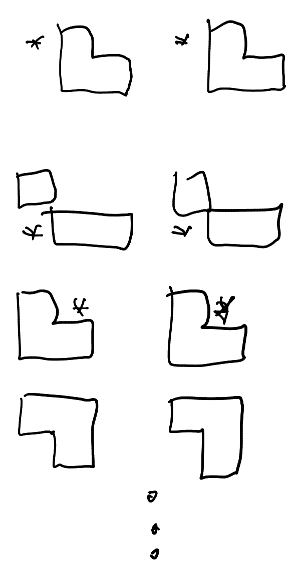
$V: js(T) = V: js(T')$

$(js(T) \sim js(T'))$

$j^T(S) = j^{T'}(S)$

Dual equivalence: $S \sim T$ if can't distinguish them by shapes, and changes of shape under slides.



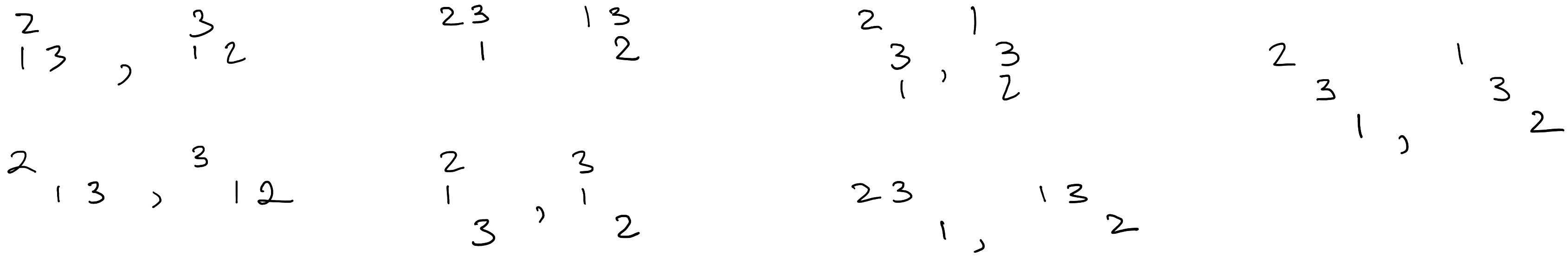


S, S':

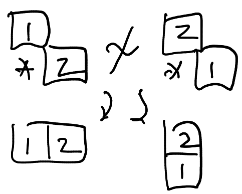
$V: j^T(S) = V: j^T(S')$

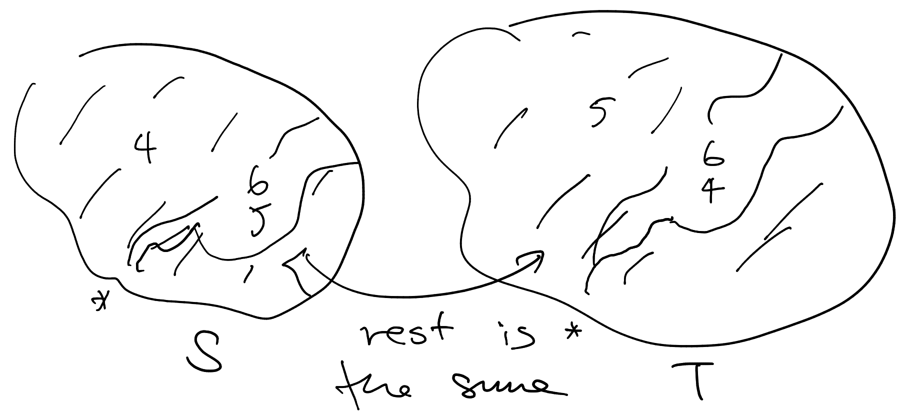
$||$ $\qquad$ $||$

$js(T)$ $\qquad$ $js'(T)$



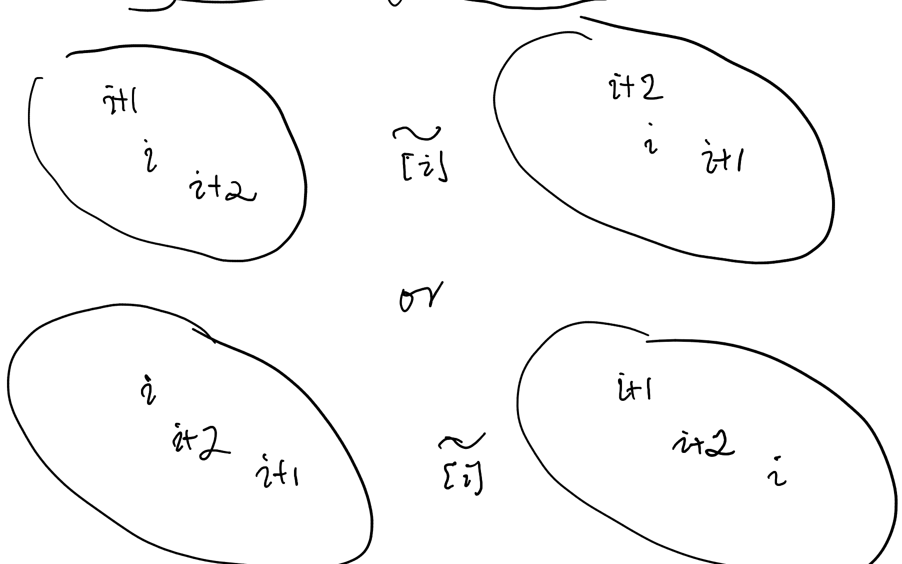
This list is closed under slides $\Rightarrow$ $\begin{smallmatrix}2\\1&3\end{smallmatrix} \sim \begin{smallmatrix}3\\1&2\end{smallmatrix}$ (and every pair on the list is a dual equivalence)



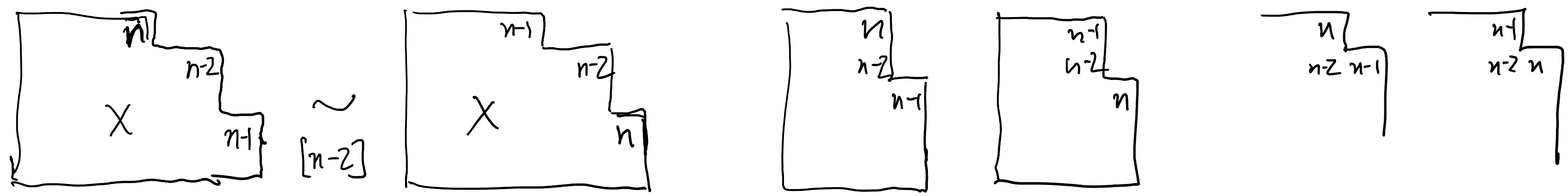


$\Rightarrow S \sim T$

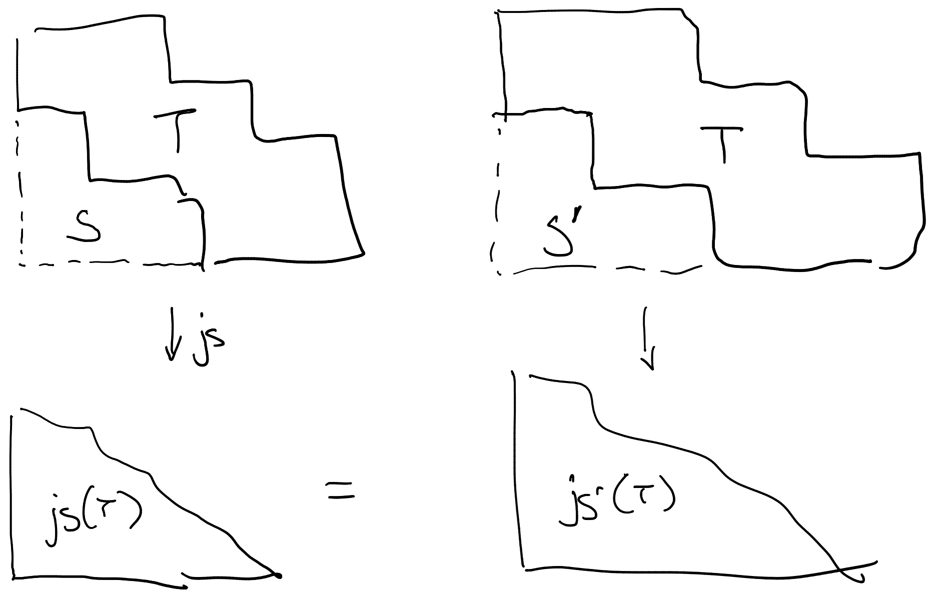
$S \sim T$ is an <u>elementary dual equivalence</u>



Prop:
λ straight shape $\Rightarrow$ all $T \in SYT(\lambda)$ are $\sim$ to each other



$|\lambda| = n$



$S \sim S'$

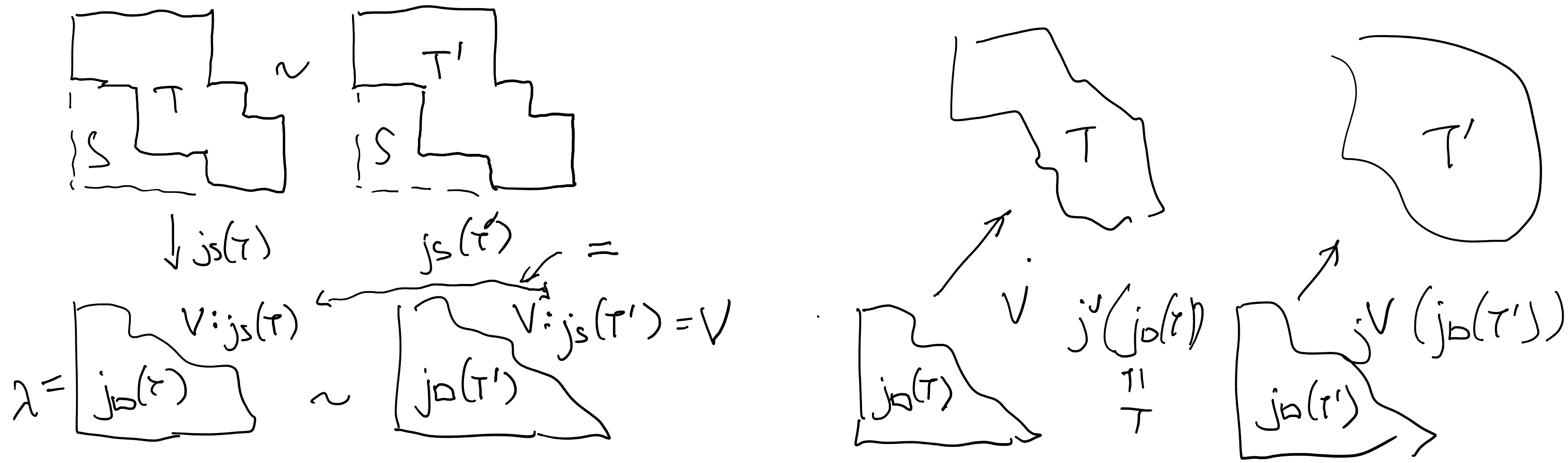
$\Downarrow$ (switching)

$js(T) = js'(T)$

Slides to straight shape give a unique tableau $j_{\square}(T)$ – the rectification of T.

```
5 8              5 8             8              8             8
1 6 7 9    →     1 6 7      →    5 6 7     →    5 6 7    →    5 6
  • 3 4          • 3 4 9         1 3 4 9        1 3 4         1 3 7
      2 10            2 10         • 2 10        • 2 9 10      • 2 4 9 10
                                                        //          ↓
5 8              5 8            5 8           5 8          8             8
1 6 7 9    →     1 6 7    →     1 6     →     1 6     →    5 6     →     5
    3 4            • 3 9          3 7 9       • 3 7        1 3 7         3 6 7
    • 2 10          2 4 10        • 2 4 10      2 4 9 10   • 2 4 9 10    1 2 4 9 10
```

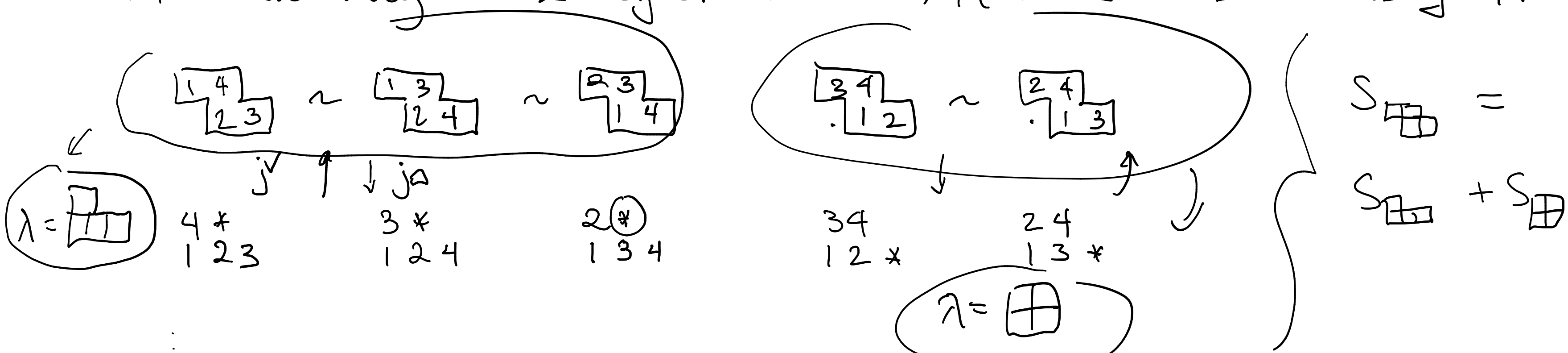
1st Fund. of Jeu-de-Taquin (uniqueness of rectification).



$j\Delta(-) = js(-) \longrightarrow$

$\longleftarrow \; j^V(-)$

Are mutually inverse bijections $SYT(\lambda) \rightleftarrows$ D.e. class of T.



2nd Fund. Thm $|\{T \in SYT(\nu/\mu) : j\Delta(T) = S\}|$ depends only on shape λ of S.

given, straight shape

$= \#$ DE classes in $SYT(\nu/\mu)$ that rectify to $SYT(\lambda)$.

$\Rightarrow$ 1. Rectification of an SSYT is unique.

2. $|\{T \in SSYT(\nu/\mu)\} : jdt(T) = S\}|$ depends only on λ.

S given, $\in SSYT(\lambda)$

$\# = C^{\lambda}_{\nu/\mu}$ Littlewood-Richardson

$$\Rightarrow \sum_{T \in SSYT(\nu/\mu)} x^T = s_{\nu/\mu}$$

$$\big\downarrow \; jdt$$

$$\sum_{\lambda} C^{\lambda}_{\nu/\mu} \sum_{T \in SSYT(\lambda)} x^T = \sum_{\lambda} C^{\lambda}_{\nu/\mu} s_{\lambda}$$

$$C^{\lambda}_{\nu/\mu} = c^{\nu}_{\lambda\mu}$$

$$s_{(3,2)/(1)} = s_{(3,1)} + s_{(2,2)}$$

$\nu = (3,2)$

$\mu = (1)$

$$s_{\square} \cdot s_{(3,1)} = s_{(3,1,1)} + s_{(3,2)} + s_{(4,1)}$$

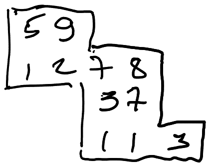
$$s_{\square} \cdot s_{(2,2)} = s_{(2,2,1)} + s_{(3,2)}$$

$$c^{(3,2)}_{\square,(3,1)} = 1 = c^{(3,1)}_{(3,2)/\square}$$

$$c^{(3,2)}_{\square,(3,1)} = c^{\square}_{(3,2)/(3,1)} = c^{\triangle}_{\triangle}$$

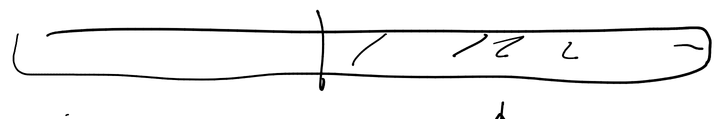
$$c^{(3,2)}_{\square,(2,2)} = 1 = c^{(2,2)}_{(3,2)/\square}$$

$$c^{(3,2)}_{\square,(2,2)} = c^{\square}_{(3,2)/(2,2)} = c^{\square}_{\square} = 1$$



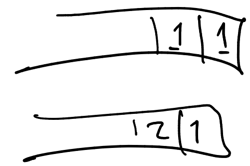
reading word $\longrightarrow$ 5 9 1 2 7 8 3 7 1 1 3

A Word is <u>Yamanouchi</u> if · weight is a partition (#1's ≥ #2's ≥ ⋯) and this holds in every tail of the word.

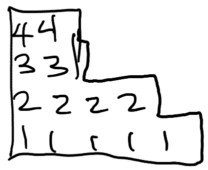


#1's ≥ #2's

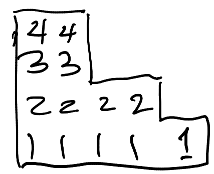
#2's ≥ #3's



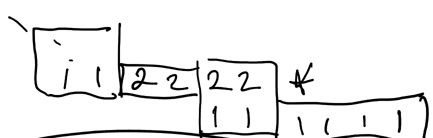
Def. Tableau is Yam. if its reading word is Yam.

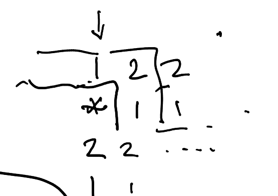
λ  $\longrightarrow$ 4 4 3 3 2 2 2 2 1 1 1 1 1 is Yamanouchi.

Only Yamanouchi of shape λ!



J-d-T slides preserve Yamanouchi tableaux.





$c^{\lambda}_{\nu/\mu}$ = # Yamanouchi SSYT of shape ν/μ and weight λ ← classical Littlewood Richardson.